

Lattice QCD Study for Confinement in Hadrons

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We study three subjects on quark confinement in hadrons in $SU(3)_c$ lattice QCD. From the accurate lattice calculation for more than 300 different patterns of three-quark (3Q) systems, we find that the static 3Q potential is well described by Y-Ansatz, i.e., the Coulomb plus Y-type linear potential. We also study the multi-quark (4Q, 5Q) potentials in lattice QCD, and find that they are well described by the one-gluon-exchange (OGE) Coulomb plus string-theoretical linear potential, which supports the *infrared string picture* even for the multi-quarks. The second subject is a lattice-QCD determination of the relevant gluonic momentum component for confinement. The string tension (confining force) is found to be almost unchanged even after cutting off the high-momentum gluon component above 1.5GeV in the Landau gauge. In fact, *quark confinement originates from the low-momentum gluon below about 1.5GeV*. Finally, we consider a possible gauge of QCD for the quark potential model, by investigating “instantaneous inter-quark potential” in generalized Landau gauge, which describes a continuous change from the Landau gauge to the Coulomb gauge.

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I. INTRODUCTION

In 1966, Yoichiro Nambu [1] first proposed the $SU(3)_c$ gauge theory, i.e., quantum chromodynamics (QCD), as a candidate for the fundamental theory of the strong interaction, just after the introduction of “color” [2]. Around 1970, he also proposed the string theory for hadrons [3], which describes hadron phenomenology. In 1973, the asymptotic freedom of QCD was theoretically shown [4], and QCD was established as the fundamental theory of the strong interaction. However, in spite of its simple form, QCD creates thousands of hadrons and leads to various interesting nonperturbative phenomena such as color confinement and chiral symmetry breaking [5]. Even now, it is very difficult to deal with QCD analytically due to its strong-coupling nature at the low-energy. Instead, lattice QCD is now a reliable numerical method to analyze nonperturbative QCD. In this paper, we study three subjects on quark confinement in hadrons in $SU(3)_c$ lattice QCD.

II. INFRARED STRING PICTURE FOR BARYONS/MULTI-QUARKS

Around 1980, the first application of lattice QCD Monte Carlo simulations [6] was done by M. Creutz for the static quark-antiquark ($Q\bar{Q}$) potential. Since then, the study of inter-quark forces has been one of the important issues in lattice QCD [7]. Actually, in hadron physics, the inter-quark force can be regarded as an elementary quantity to connect the “quark world” to the “hadron world”, and plays an important role to hadron properties.

Around 2000, we performed the first accurate reliable lattice QCD study for the three-quark (3Q) potential, which is responsible to the baryon structure at the quark-gluon level. Furthermore, in 2005, we performed the first lattice QCD study for the multi-quark potentials, i.e., 4Q and 5Q potentials, which give essential information for the multi-quark hadron physics. Note also that *the studies of 3Q and multi-quark potentials are directly related to the quark confinement properties in baryons and multi-quark hadrons*.

Here, we summarize our detailed studies of the inter-quark forces in three-quark/ multi-quark systems with $SU(3)_c$ quenched lattice QCD [8–10].

In lattice QCD, the static $Q\bar{Q}$ potential $V_{Q\bar{Q}}(r)$ is calculated with the Wilson loop, and is well described with the Coulomb plus linear form as [7, 8]

$$V_{Q\bar{Q}}(r) = -\frac{A_{Q\bar{Q}}}{r} + \sigma_{Q\bar{Q}}r + C_{Q\bar{Q}}. \quad (1)$$

This physically means one-dimensional flux-tube formation between quark and antiquark, and this one-dimensional squeezing of color flux is shown in lattice QCD [7].

For the color-singlet baryonic 3Q system, the 3Q potential can be calculated with the 3Q Wilson loop [8]. For more than 300 different patterns of 3Q systems, we perform accurate calculation of the static 3Q potential V_{3Q} in lattice QCD on various lattices: $(\beta=5.7, 12^3 \times 24)$, $(\beta=5.8, 16^3 \times 32)$, $(\beta=6.0, 16^3 \times 32)$, $(\beta=6.2, 24^4)$. We find that V_{3Q} is well described by the Coulomb plus Y-type linear potential (Y-Ansatz) [8–10],

$$V_{3Q} = -A_{3Q} \sum_{i < j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{3Q} L_{\min} + C_{3Q}, \quad (2)$$

where L_{\min} is the minimal total length of the color flux tube linking the quarks, i.e., the Y-shaped flux-tube length in most cases. As an example, we show in Fig.1(a) the 3Q confinement potential V_{3Q}^{conf} , i.e., the 3Q potential subtracted by the Coulomb part, plotted against the Y-shaped flux-tube length L_{\min} . At each β , clear linear correspondence is found between V_{3Q}^{conf} and L_{\min} , which indicates Y-Ansatz. This physically means Y-shaped flux-tube formation among three quarks, and the Y-type flux-tube formation is actually observed in lattice QCD from the measurement of the action density in the spatially-fixed 3Q system [11], as shown in Fig.1(b). Y-Ansatz has been also supported by many other recent studies in lattice QCD [12] and theories including AdS/CFT [13].

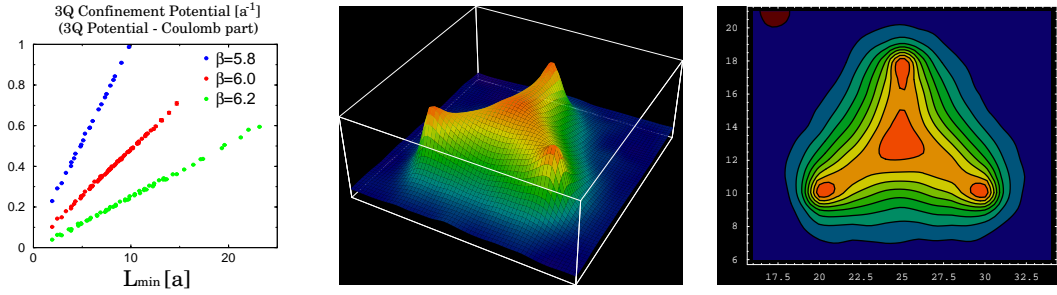


FIG. 1: (a) The 3Q confinement potential V_{3Q}^{conf} , i.e., the 3Q potential subtracted by the Coulomb part, plotted against L_{\min} of Y-Ansatz in lattice unit [9]. (b) The lattice QCD result for Y-type flux-tube formation in a spatially-fixed 3Q system [11]. The distance of each quark from the junction is about 0.5fm.

After the experimental report of penta-quark candidates, lots of theoretical analyses for the exotic hadrons have been done or revisited. Recently, several charmed tetra-quark candidates such as $X(3872)$ and $Z^+(4430)$ have been experimentally discovered, and tetra-quark has been also investigated as an interesting object in quark-hadron physics.

Obviously, the quark-model calculation is one of the standard theoretical methods to investigate multi-quark systems. In fact, the quark model calculation gives an outline of the properties of multi-quark hadrons and may predict new-type exotic hadrons theoretically. However, for such calculations, one needs the quark-model Hamiltonian for the multi-quark system. In particular, one has to know the quark confining potential in multi-quarks. Then, we performed the first lattice QCD study of multi-quark potential.

We formulate the multi-quark Wilson loop, and accurately calculate the multi-quark potential in lattice QCD for about 200 different multi-quarks [9]. We find that the multi-quark potential obeys the OGE Coulomb plus string-theoretical linear potential [9],

$$V_{nQ} = \frac{3}{2} A_{nQ} \sum_{i < j} \frac{T_i^a T_j^a}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{nQ} L_{\min} + C_{nQ} \quad (n = 3, 4, 5, \dots), \quad (3)$$

where the confinement potential is proportional to the minimal total length L_{\min} of the color flux tube linking the quarks. (Figure 2(a) and (b) are examples of the minimal-length flux-tube for the 4Q system.) We find the *universality of the string tension*, $\sigma_{nQ} \simeq \sigma_{Q\bar{Q}}$, and the OGE result, $A_{nQ} \simeq A_{Q\bar{Q}}/2$, for $n = 3, 4, 5$ [8, 9].

For the multi-quark system, the topology of the flux-tube or the string linking quarks can be changed, according to the quark location. We also investigate the flux-tube recombination for 4Q systems [9]. As the example, we show in Fig.2(c) the lattice QCD result of the 4Q potential V_{4Q} for rectangular 4Q configurations as shown in Figs.2(a) and (b), with $d \equiv \overline{Q_1 Q_2}/2$ and $h \equiv \overline{Q_1 Q_3}$. For large h , the lattice data obey the connected 4Q state energy. For small h , the lattice data obey the “two-meson” state energy, i.e., $2V_{Q\bar{Q}}(h)$. In fact, the 4Q potential V_{4Q} is found to take the smaller energy of the connected 4Q state or the two-meson state [9]. In other words, we observe a clear lattice QCD evidence of the “flip-flop”, i.e., the string recombination between the connected 4Q state and the two-meson state.

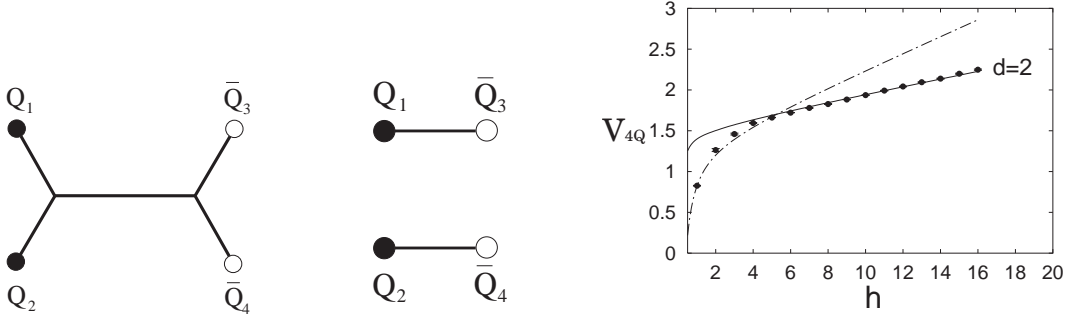


FIG. 2: (a) A connected 4Q state and (b) a “two-meson” state. (c) An example of the lattice QCD result of the 4Q potential V_{4Q} for rectangular 4Q configurations in lattice unit [9]. The symbols denote the lattice QCD data. The solid (dotted-dashed) curve denotes connected 4Q (two-meson) state energy.

In this way, our lattice results physically indicates the validity of *infrared string picture*, not only for mesons but also for baryons and multi-quarks.

Then, the multi-quark Hamiltonian based on lattice QCD is expressed as

$$H_{nQ} = \sum_{i=1}^n (\hat{\mathbf{p}}_i^2 + M_i^2)^{1/2} + \sum_{i<j}^n V_{\text{OGE}}(\mathbf{r}_i - \mathbf{r}_j) + \sigma L_{\min} \quad (n = 3, 4, 5, \dots), \quad (4)$$

where V_{OGE} is the two-body OGE potential including the spin-dependent part, and M_i the constituent quark mass. The confinement part is described by infrared string picture.

As other recent developments of multi-quark/gluon potentials in lattice QCD, we studied the excited-state 3Q potential [10] and the heavy-heavy-light quark (QQq) potential [14], and Cardoso-Bicudo studied the three-gluon static potential [15].

III. RELEVANT MOMENTUM OF GLUON FOR CONFINEMENT

Many theoretical physicists consider that confinement is brought by low-energy strong interaction of QCD. However, no one has quantitatively showed the relevant energy region for confinement directly from QCD. Here, the key question is “*What is the important energy scale for confinement in QCD?*” Since confinement is mainly brought by gluon dynamics, this can be rewritten as “*What is the relevant gluon momentum component responsible for confinement?*”

To answer this question, we formulate a new lattice method to extract relevant gluon momentum for each QCD phenomenon, based on four-dimensional (4D) discrete Fourier transformation [16]. For each gauge configuration, we select or remove gluon momentum components in lattice QCD by the following five steps.

1. *Generation of link-variable:* We generate gauge configurations on a L^4 lattice with the lattice spacing a by the lattice-QCD Monte Carlo simulation under space-time periodic boundary conditions, and obtain coordinate-space link-variables $U_\mu(x)$. We here take the Landau gauge to suppress the artificial gauge fluctuation [16, 17].
2. *Fourier transformation:* By the 4D discrete Fourier transformation, we define the “momentum-space link-variable”, $\tilde{U}_\mu(p) \equiv \frac{1}{L^4} \sum_x U_\mu(x) \exp(i \sum_\nu p_\nu x_\nu)$. The “momentum-space lattice spacing” is $a_p \equiv 2\pi/(La)$.
3. *Cut in the momentum space:* We introduce a cut (UV cut or IR cut) in the momentum space. Outside the cut, $\tilde{U}_\mu(p)$ is replaced by the free-field link-variable. We then obtain the momentum-space link-variable $\tilde{U}_\mu^\Lambda(p)$ with a cut.
4. *Inverse Fourier transformation:* To return to coordinate space, we carry out the inverse Fourier transformation. Since this $U'_\mu(x)$ is not an SU(3) matrix, we project it onto an SU(3) element $U_\mu^\Lambda(x)$ by maximizing $\text{ReTr}\{U_\mu^\Lambda(x)^\dagger U'_\mu(x)\}$. Then, we get coordinate-space link-variable $U_\mu^\Lambda(x)$ with the cut.
5. *Calculation:* Using the cut link-variable $U_\mu^\Lambda(x)$ instead of $U_\mu(x)$, we calculate physical quantities as the expectation values in the same way as ordinary lattice calculations. This procedure is applicable to all the quantities in lattice QCD.

With this method, we quantitatively determine the relevant gluon momentum component for confinement directly from QCD, through the analyses of the $Q\bar{Q}$ potential [16]. We perform quenched $SU(3)_c$ lattice QCD calculations on 16^4 lattice at $\beta=5.7, 5.8$ and 6.0 .

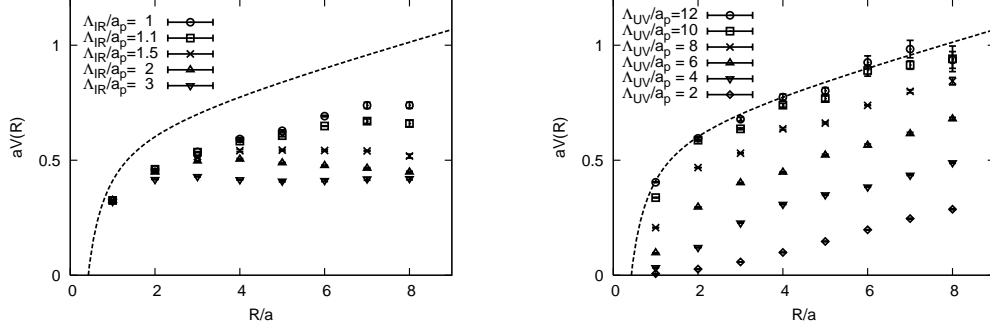


FIG. 3: (a) The $Q\bar{Q}$ potential $V(R)$ with the IR cut Λ_{IR} plotted against the inter-quark distance R . (b) The $Q\bar{Q}$ potential with the UV cut Λ_{UV} . The lattice QCD calculation is performed on 16^4 lattice with $\beta = 6.0$, *i.e.*, $a \simeq 0.10\text{fm}$ and $a_p \equiv 2\pi/(La) \simeq 0.77\text{GeV}$. The broken line is the original $Q\bar{Q}$ potential.

Figure 3 (a) and (b) show the $Q\bar{Q}$ potential $V(R)$ with the IR cutoff Λ_{IR} and the UV cutoff Λ_{UV} , respectively [16]. Then, we obtain the following lattice-QCD results about the role of gluon momentum components on the inter-quark potential.

- By the IR cutoff Λ_{IR} , as shown in Fig.3(a), the Coulomb potential seems to be unchanged, but the confinement potential is largely reduced.
- By the UV cutoff Λ_{UV} , as shown in Fig.3(b), the Coulomb potential is largely reduced, but the confinement potential is almost unchanged.

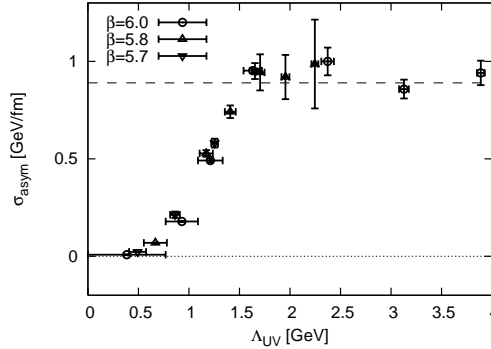


FIG. 4: The Λ_{UV} -dependence of the string tension $\sigma(\Lambda_{UV})$ obtained from the asymptotic slope of the $Q\bar{Q}$ potential $V(R)$ with the UV cutoff Λ_{UV} . The lattice QCD calculations are done on 16^4 lattice with $\beta = 5.7, 5.8$ and 6.0 . The vertical error-bar is the statistical error, and the horizontal one is the range from the discrete momentum. The broken line denotes the original value of the string tension $\sigma \simeq 0.89\text{GeV/fm}$.

Figure 4 shows the Λ_{UV} -dependence of the string tension $\sigma(\Lambda_{UV})$ obtained from the asymptotic slope of the $Q\bar{Q}$ potential $V(R)$ with the UV cutoff Λ_{UV} in the Landau gauge [16]. (Similar results are obtained also in the Coulomb gauge.) As a remarkable fact, the string tension (confining force) is almost unchanged even after cutting off the high-momentum gluon component above 1.5GeV , while the string tension is significantly reduced when the UV cutoff is smaller than about 1.5GeV . We thus conclude that *quark confinement originates from the low-momentum gluon component below about 1.5GeV* [16].

IV. ATTEMPT OF LINKAGE FROM QCD TO QUARK MODEL

While QCD is the fundamental gauge theory of strong interaction, the quark model is also a successful model in describing hadrons. However, their relation is still unclear. We here consider two paths from QCD to the quark

model, based on lattice-QCD results.

A. Large Gluonic-Excitation Energy and Success of the Quark Model

In the quark model, most low-lying hadrons can be well described only with quark degrees of freedom. *Why dynamical gluons do not appear in low-lying hadrons?*

We think that the absence of dynamical gluons is due to the large gluonic-excitation energy of about 1GeV for mesons [18] and baryons [10]. (See Fig.5.) This is much larger than the quark-origin excitation such as spin-dependent interaction. Then, for low-lying hadrons, the gluonic excitation is absent, and the system can be expressed only with quark degrees of freedom, which gives a background of the success of the quark model. [In terms of the gluonic excitation, the hybrid hadron appears as a higher-excited state of about 1GeV, which explains the mass of Y(3940) as $4\text{GeV} \simeq 1.5\text{GeV} \times 2 + 1\text{GeV}$.]

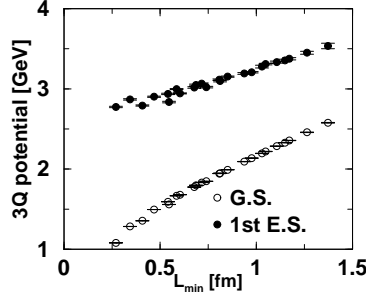


FIG. 5: The 1st excited-state 3Q potential $V_{3Q}^{e.s.}$ (filled circle) and the ground-state 3Q potential $V_{3Q}^{g.s.}$ (open circle) v.s. L_{\min} in lattice QCD at $\beta = 5.8$ [10]. $\Delta E_{3Q} \equiv V_{3Q}^{e.s.} - V_{3Q}^{g.s.}$ is the gluonic-excitation energy.

B. What is the Gauge of QCD for the Quark Potential Model?

The quark potential model is a nonrelativistic model with a potential instantaneously acting among quarks. In this model, there are no dynamical gluons, and gluonic effects indirectly appear as the instantaneous inter-quark potential. From the viewpoint of “gauge” in QCD, the quark model without dynamical gluons can be regarded as an effective theory of gauge-fixed QCD. Since the quark model has rotational and global color symmetries, such a gauge should keep them like Landau and Coulomb gauges.

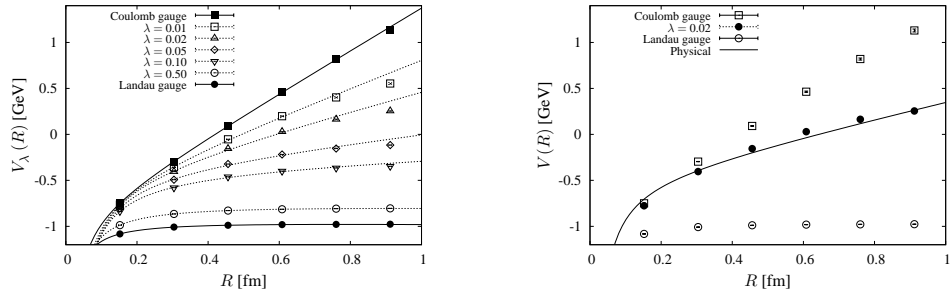


FIG. 6: (a) “Instantaneous potential” $V_\lambda(R)$ in generalized Landau gauge for typical values of λ . The symbols are the lattice QCD results, and each curve denotes the fit with the Coulomb plus linear form. (b) Comparison of the instantaneous potential $V_\lambda(R)$ at $\lambda = 0.02 (\simeq \lambda_C)$ (black dots) with the physical static inter-quark potential (line). We add the lattice data in Coulomb ($\lambda = 0$) and Landau ($\lambda = 1$) gauges.

With a real parameter $\lambda \geq 0$, we define generalized Landau gauge (λ -gauge) [19] of $\partial_i A_i + \lambda \partial_4 A_4 = 0$. This gauge can connect continuously between the Landau ($\lambda=1$) and the Coulomb ($\lambda=0$) gauges. In lattice QCD, we investigate “instantaneous potential” $V_\lambda(R) \equiv -\frac{1}{a} \ln \langle \text{Tr}[U_4^\dagger(\mathbf{R}, t) U_4(\mathbf{0}, t)] \rangle$ in λ -gauge [19]. Figure 6 shows the lattice result of the instantaneous potential [19]: no linear part appears in the Landau gauge ($\lambda=1$); the Coulomb gauge ($\lambda=0$) shows

overconfining with $2\sim 3$ times larger confining force [20], so that the ground state is conjectured to be the gluon-chain state [21]. In the λ_C -gauge with $\lambda_C \simeq 0.02$, the physical static inter-quark potential is approximately reproduced by the instantaneous potential [19], which seems to match the quark potential model.

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